MATH 1730

2.4 - Abs maxima and minima
1. find f'(x)
2. find any critical points
3. if looking over a closed interval, use 1st order derivative
4. if over an open inverval/all values, use 2nd order.

1 $f(x) = x^3 - 3x + 16$ $[-1, 3]$

$f'(x) = 0$

$f'(x) = 3x^2 - 3$

$f'(x) = 0$

$\Rightarrow 3x^2 - 3 = 0$

$\div 3 \Rightarrow x^2 - 1 = 0$

$\Rightarrow (x+1)(x-1) = 0$

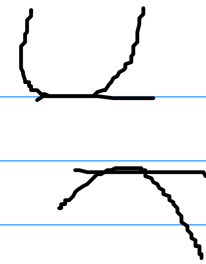
$x = 1, x = -1$

$-1 \Rightarrow f(-1) = (-1)^3 - 3(-1) + 16 = -1 + 3 + 16 = 18$

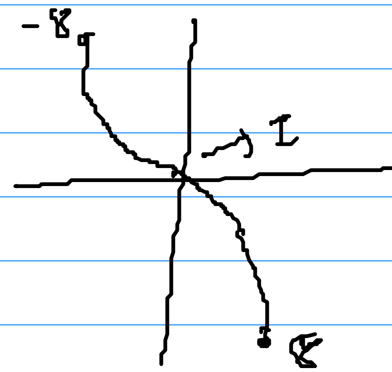
$1 \Rightarrow f(1) = 1^3 - 3 \cdot (1) + 16 = 1 - 3 + 16 = 14$

$3 \Rightarrow f(3) = 3^3 - 3(3) + 16 = 27 - 9 + 16 = 34$

$f(x) = 1 - x^3$ $\quad [-8, 8]$



$f'(x) = 3x^2$



$f'(x) = 0$

$\Rightarrow 3x^2 = 0$

$\Rightarrow \underline{x = 0}$

$f''(x) = 6x$

$< 0 \to$ max
$> 0 \to$ min

$\left.\underline{f''(0)} = 6 \cdot 0 = 0\right\}$ $= 0 \Rightarrow$ inflection

$f(-8) = 1 - (-8)^3 = 1 - (-512) = 513 \to$ max

$f(0) = 1 \to$ P.O.I.

$f(8) = 1 - 8^3 = -511 \to$ min

$$f(x) = 1 - x^{2/3} \qquad [-8, 8]$$

$$f'(x) = -\frac{2}{3} x^{2/3 - 1}$$

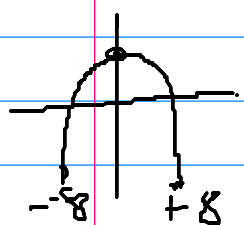
$$= -\frac{2}{3} x^{-1/3}$$

$$= -\frac{2}{3} \frac{1}{\sqrt[3]{x}}$$

$$f''(x) = -\frac{2}{3} \cdot \left(-\frac{1}{3}\right) x^{-4/3}$$

$$= \frac{2}{9} x^{-4/3}$$

$$= \frac{2}{9} \cdot \frac{1}{\sqrt[3]{x^4}}$$



$f'(x)$ d.n.e for $\underbrace{x = 0}_{\text{crit. pt.}}$

$f'(x) \neq 0$ for any x

$$+8: \; f(+8) = 1 - (8)^{2/3} = 1 - \sqrt[3]{8^2} = 1 - \sqrt[3]{64} = 1 - 4 = \underbrace{-3}_{\text{min}} = f(-8)$$

$$0: \; f(0) = 1 - \sqrt[3]{0^2} = \underbrace{1}_{\text{max}}$$

$$f(x) = (x+3)^{2/3} - 5 \quad , \quad [-4, 5]$$

$$f'(x) = \frac{2}{3}(x+3)^{-1/3} \cdot 1 \qquad \begin{array}{cc} -4 & 4 \\ \uparrow & \uparrow \end{array}$$

$$= \frac{2}{3} \frac{1}{\sqrt[3]{x+3}} \qquad \begin{pmatrix} x, f(x) \\ (x, y) \end{pmatrix}$$

-3 is a c.P $\Rightarrow$ $f'(-3)$ d.n.e

$f'(x) \neq 0$ for any x

$$f(-4) = (-4+3)^{2/3} - 5$$

$$\sqrt[3]{(-1)^2} - 5 = 1 - 5 = -4$$

$$f(-3) = \sqrt[3]{(3-3)^2} - 5 = -5 \longrightarrow \text{min}$$

$$f(5) = \sqrt[3]{(5+3)^2} - 5 = \sqrt[3]{64} - 5 = 4 - 5 = -1$$

max

$$f(x) = \frac{x^2}{x^2+1} \qquad [-2, 2]$$

$$u = x^2 \qquad v = x^2+1$$

$$u' = 2x \qquad v' = 2x$$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{2x(x^2+1) - x^2(2x)}{(x^2+1)^2}$$

$f'(x)$ exists for all x

$f'(x) = 0$

$$\Rightarrow \frac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2} = 0$$

$x(x^2+1)^2$ $\Rightarrow 2x(x^2+1) - 2x \cdot x^2 = 0$

$$\Rightarrow 2x(\cancel{x^2}+1-\cancel{x^2}) = 0$$

$$\Rightarrow 2x = 0 \Rightarrow x = 0$$

c.p at $x = 0$

$$f(x) = \frac{x^2}{x^2+1}$$

$-2 \quad f(-2) = \frac{4}{4+1} = \frac{4}{5} \longrightarrow max$

$0 \quad f(0) = \frac{0}{0+1} = \frac{0}{1} = 0 \longrightarrow min$

$2 \quad f(2) = \frac{4}{5} \longrightarrow max$

$$f(x) = 2x + \frac{72}{x} \qquad (0, \infty)$$

$$f'(x) = 2 - \frac{72}{x^2}$$

~~$f'(x)$ d.n.e. for $x=0$~~

$$f'(x) = 0$$

$$\Rightarrow 2 - \frac{72}{x^2} = 0$$

$$\Rightarrow 2 = \frac{72}{x^2} \Rightarrow x^2 = \frac{72}{2}$$

$$\Rightarrow x^2 = 36 \Rightarrow x = 6$$

$$f'(x) = 2 - \frac{72}{x^2}$$

$$f''(x) = -(-2)\cdot\frac{72}{x^3}$$

$$= \frac{2\cdot 72}{x^3}$$

$$f''(6) = \frac{2\times 72}{6^3} = \frac{2\times \cancel{72}^{\,2}}{\cancel{36}\times\cancel{6}_{\,3}} = \frac{2}{3} > 0$$

$$\|$$

$$\frac{2\times \cancel{72}^{\,\cancel{12}\ \cancel{2}}}{\cancel{6}\times 6\times\cancel{6}_{\,3}} = \frac{2}{3} > 0 \rightarrow \text{min}$$

Revenue

~~price~~ per object = 280 - .4x
C(x) = 5000 + 0.6*x^2

Maximise profit

steps:
1. find revenue fn (total revenue fn)
2. find cost fn.
3. profit = R(x) - C(x)
4. take derivative and find maxima

$$R(x) = (\underbrace{280 - 0.4x}_{\text{/object}})\underbrace{x}_{\text{total \#}}$$

$$C(x) = 5000 + 0.6x^2$$

$$P(x) = (280 - 0.4x)x - 5000 + 0.6x^2$$

$P'(x) =$

$P'(x) = 0$

price per suit = 150 - 0.5x
total cost of producing suits = 4000 + .25x^2

1. find total revenue
2. find total profit
3. maximise profit

1 total price = total revenue = (150-.5x)*x

2 total profit = revenue - cost (remember that all values are totals)

= (150-.5x)x - (4000 + .25x^2)

$$P = 150x - \frac{1}{2}x^2 - 4000 - \frac{1}{4}x^2$$

$$P'(x) = \overline{150} - 2\cdot\frac{1}{2}x - \frac{2}{4}x$$

$$P'(x) = 0 \Rightarrow 150 - x - \frac{x}{2} = 0$$

$$\Rightarrow 150 = \frac{3x}{2} \Rightarrow x = \overset{50}{\cancel{150}} \times \frac{2}{\cancel{3}} = 100$$

$\downarrow$

# of suits

$$\text{max profit} = 150 \times 100 - 0.75 \times 100^2 - 4000$$

$$= 11000 - 0.75 \times 10000$$

$$= 10000 - 7500$$

$$= 3500$$

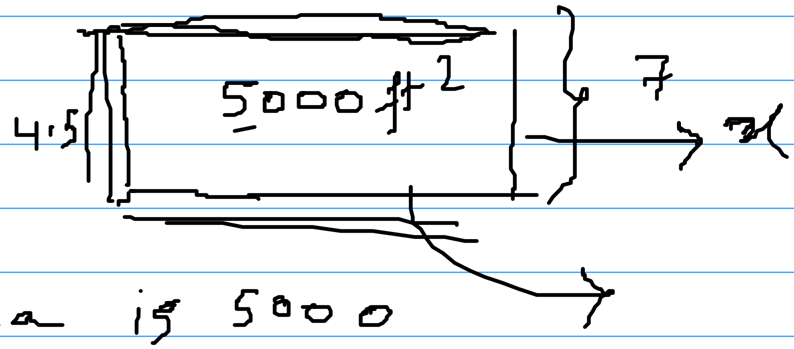
minimising cost:

rectangular parking lot with area 5000 sq. ft which needs to be fenced

enclosed on 3 sides with fencing that costs $4.5/ft

the 4th side will have a wooden fence that costs $7/ft

what should be the dimensions of the field so that the fencing cost is minimised



if area is 5000

$$x \times (\_\_) = 5000$$

the sides are x, $\frac{5000}{x}$

$$4.5 \times \frac{5000}{x} \times 2 + 4.5x + 7x = C(x)$$

$$\frac{9 \times 5000}{x} + 11.5x$$

$$c'(x) = 11.5 - \frac{9 \times 5000}{x^2}$$

$c'(x) = 0$

$\Rightarrow 11.5 - \frac{5000 \times 9}{x^2} = 0$

$\Rightarrow x^2 = \frac{5000 \times 9}{11.5}$

$\Rightarrow x = \sqrt{\frac{45000}{11.5}} = \underline{62.55}$

$\frac{5000}{x} = \underline{79.93}$